$1\ kpc = 3 \cdot 10^{19}\ m$

$1\ \mu G = 10^{-10}\ T$

$e = 1.6 \cdot 10^{-19}\ C$

$\mu = 1.7 \cdot 10^{-27}\ kg$

NAASPH-12.2017-2018.2A Exam April 4 2018

Astroparticle Physics

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Write your name and S number on every sheet
You don't have to use separate sheets for every question.
There are **6 questions** with a total number of marks: 37

WRITE CLEARLY

(1) (Total 8 marks)
In the framework of the first-order Fermi mechanism, the acceleration process of charged particles is carried out in shock fronts which occur, for example, in supernovae.

(a) (1 mark)
Suppose that the energy gain per cycle is $\Delta E = \epsilon E_0$, where E_0 is the particle initial energy. What is the energy of the particle after it has passed the cycle n times?

(b) (1 mark)
The probability that a particle is confined in the accelerator region is P_c per cycle. How many of N_0 initial particles have still remained in the acceleration process after n cycles?

(c) (2 marks)
Determine the energy spectrum $\frac{dN}{dE}$ of all particles that have undergone the acceleration process, depending on the initial energy E_0.

(d) (1 mark)
Typical velocities of shock fronts from supernovae are $v_S \simeq 6\ 10^4$ km/s. Compute the spectral index of the energy spectrum if the probability that a particle remains confined is $P_c = 70\%$.

(e) (1 mark)
For a particle of initial energy $E_0 = 1$ MeV, how many times does it have to go through the shock front until it reaches an energy of 1 TeV ?

(f) (2 marks)
Demonstrate that the Larmor radius r_L of a particle of charge Ze and energy E in a magnetic field of strength B is given by $r_L = \frac{E}{ZeB}$ and use this expression to derive the maximal energy that can be reached by a carbon atom ($Z = 6$) being accelerated in a galaxy with size $L = 30$ kpc and magnetic field $B = 1\ \mu$G.

(2) (Total 6 marks)
In a simplified model the development of the shower depends on the initial energy of the primary particle E_0, as well as the critical energy E_c, that is a property of the medium where the shower develops. For electromagnetic showers in air $E_c = 100$ MeV.

(a) (2 marks)
Compute the maximum number of particles in the shower generated by a photon of initial energy $E_0 =$ 1 TeV, entering the atmosphere vertically.

(b) (1 mark)
How is the critical energy defined ?

(c) (1 mark)
For a vertical shower, the depth x in the upper atmosphere (in g/cm^2) can be expressed as a function of height h using the following expression:

$$x \propto X e^{-h/H}$$

where $X \simeq 1030$ g/cm^2, $H = 7$ km. Consider $X_0(air) = 36.6$ g/cm^2.
Calculate the maximum depth height h_{max} in the Heitler model.

(d) (2 marks)
Define the concept of the light pool, and compute the area of the light pool assuming the Cherenkov light to be emitted at height h_{max} with an aperture angle of 1.3 degrees.

(3) (Total 3 marks)
Why is the sky dark at night and not as bright as during daylight ? This question was asked by Heinrich Wilhelm Olbers (1758 - 1840), a German astronomer. He assumed that the cosmological principle is valid. He realized that the light intensity drops by r^{-2} and that at the same time the number of stars in a shell with thickness dr with a radius r increases as $4\pi r^2 dr$. Therefore, every portion of the sky must end up at a star, so there cannot be "darkness". This is known as the "Olbers paradox".

(a) (2 marks)
Discuss the solution to this paradox.

(b) (1 mark)
Try to help Olbers: might he have been right after all if he would have known about the cosmic microwave background radiation?

(4) (Total 7 marks)
Anti-protons constitute a sub-dominant component of cosmic rays, as they are mainly produced as a consequence of primary cosmic rays interactions with the interstellar medium (ISM). Cosmic-ray anti-proton production takes place via the following process:

$$p + p_{ISM} \rightarrow \bar{p} + p + p + p$$

(a) (1 mark)
Compute the 4-momentum squared in the laboratory frame for the initial state, assuming the target, p_{ISM}, at rest.

(b) (1 marks)
Compute the 4-momentum squared for the final state in the center of mass frame, as a function of the proton mass m_p, under the hypothesis that the particles are produced at rest.

(c) (2 marks)
Compute the threshold energy of the primary (high-energy) proton for this process, as a function of the proton mass m_p.

(d) (1 mark)
Compute the mean free path of the high-energy protons, assuming that the only relevant phenomena that affects their propagation in the galaxy is the interaction with the ISM, whose density is n. The interaction cross section is given by $\sigma = 100$ mb. Consider the following numerical values: $n = 1\ cm^{-3}$, 1 mb $= 10^{-31}$ m^2.

(e) (2 marks)
Discuss the validity of the assumption made in sub-question (d), for the mean free path of protons in the galaxy. Discuss whether or not the same assumption is also valid for electrons.

(5) (Total 5 marks)
Neutrinos are elusive particles with no electric charge and very small mass compared to the charged leptons. They are created through the weak-interaction process.

(a) (1 mark)
What are the names of the force carriers for this process.

(b) (1 mark)
After traveling a distance L, a neutrino with an energy E has a certain probability to change its flavor state, e.g. from the muon to the tau flavor. In the atmosphere many muon neutrinos are created. Name the reaction(s) that describe the creation of the muon neutrinos in the atmosphere.

(c) (2 mark)
In a two-flavor scenario the appearance probability of a tau neutrino starting as a muon neutrino is given as:

$$P_{\nu_\mu \to \nu_\tau}(t) = | < \nu_\mu | \nu_\tau(t) > |^2 = \sin^2(2\theta_{23}) \sin^2[(E_2 - E_3)t/(2\hbar)]$$

where θ_{23} is the so-called mixing angle, and $E_{2,3}$ are the energies of the two mass eigenstates 2 and 3. Assuming that only the mass of these two eigenstates are different and not their momenta, proof that one can write the appearance probability as:

$$P_{\nu_\mu \to \nu_\tau}(t) = \sin^2(2\theta_{23}) \sin^2 \left[\frac{\Delta m^2 c^4}{4\hbar c} \frac{L}{E} \right]$$

where $\Delta m^2 c^4 = m_2^2 c^4 - m_3^2 c^4$.

(d) (1 mark)
Calculate the appearance probability of 1 GeV muon neutrino to be detected as a tau neutrino after passing through the Earth (diameter 13800 km). Use for the mixing angle $\theta_{23} = 45°$.
Use $\frac{1}{4\hbar c} = 1.27 \times 10^9$ eV^{-1}km^{-1} and $\Delta m^2 c^4 = 4.6 \times 10^{-5}$ eV^2.

(6) (Total 8 marks)
Charged particles loose energy when they travel through space.

(a) (2 marks)
Provide the important parameters for the energy loss mechanism in case these particles are traveling along magnetic field lines.

(b) (2 marks)
Describe the observational evidence that this mechanism is at work in the universe.

(c) (2 marks)
Provide the important parameters for the energy loss mechanism in case these particles are traveling through (intense) photon fields.

(d) (2 marks)
Describe the possible experimental evidence that this mechanism is at work in the universe.